SOCIETE FRANCAISE DE LOGIQUE
METHODOLOGIE,
ET PHILOSOPHIE DES SCIENCES

Mathématiques 2e cycle
Université Claude Bernard
43, boulevard du 11 novembre 1918
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BULLETIN D'INFORMATION N° 6

FEVRIER 1979

SOMMAIRE:

DOCUMENTS JOINTS AU BULLETIN D'INFORMATION N° 6 :

Compte-rendu de l'enquête sur la situation de l'enseignement de la logique en France.

Questionnaire à remplir et à retourner à la Société Française de Logique, Méthodologie, et Philosophie des Sciences.

SEJOURS D'ETRANGERS EN FRANCE:

- M. KECHRIS est à Paris jusqu'à fin avril.
- B. et M. MOSS seront à l'université de Clermont II du 23 au 28 avril 1979.
- C. RAUSZER sera à l'université de Clermont II du 23 au 28 avril 1979.
- C. RAUSZER sera à l'université de Lyon I du 1er au 8 mai 1979.

EXPOSES:

Le texte de la conférence donnée par R. MAGARI dans le cadre de la Table Ronde des 16 et 17 septembre 1978 (Paris) sera diffusé ultérieurement.

CLAUSES AND CLASSES

W.V. Quine

I propose to begin with a very weak logic and to augment it by a series of uniform steps. Each step may be briefly characterized as consisting in the admitting of relative clauses into categorical contexts. The first such step yields the monadic predicate calculus. The next yields the theory of classes of individuals. The next yields the theory of classes of such classes; and so on up through the theory of types.

One merit of the plan is that it puts the bound variable in the proper light, as primarily a relative pronoun used in deriving relative clauses from sentences. Another merit is that it pinpoints the transition from logic to set theory, narrowing the difference down to something seemingly rather slight and subtle; and similarly for each succeeding step up the hierarchy of types. The theory of types comes thereby to seem somewhat more natural, less arbitrary, than it otherwise may.

In my weak initial logic I use '*a*', '*b*', ... to stand for unspecified names and '*F*', '*G*', ... for unspecified predicables.[1] I admit two copulas, '$\in$' and '$\subseteq$': singular and plural. Notational these suggest membership and inclusion, and in due course the suggestion will be sustained; but at the present stage it must be resisted. When Peano introduced '$\in$' into logic and set theory he meant it as the initial of the Greek copula ἐστί, and this is how I shall now use it; '*a* $\in$ *F*' is simply the singular predication to be read '*a* is an *F*'. Correspondingly '*F* $\subseteq$ *G*' is to be read as the plural predication '*F* are *G*', 'All *F* are *G*', the universal affirmative categorical.

Such, then, are the atomic formulas of my weak logic: singular and plural predications. The rest of the formulas are the truth functions of these. A decision procedure for validity is ready to hand. The properties of the plural copula, to begin with, are simply those of a partial ordering, and so are summed up in the axiom schemata:

(1) $\underline{F} \subseteq \underline{F}$,

(2) $\underline{F} \subseteq \underline{G} \,.\, \underline{G} \subseteq \underline{H} \,.\!\supset\!.\, \underline{F} \subseteq \underline{H}$.

Of the singular copula less can be said at this stage. Reflecting that in set theory '$\underline{a} \in$' would amount to '$\{\underline{a}\} \subseteq$', we see that '$\underline{a} \in$' can supplant '$\underline{F} \subseteq$' in all laws in which '$\underline{F}$' occurs only before '$\subseteq$'. Substituting accordingly in (2), we have:

(3) $\underline{a} \in \underline{G} \,.\, \underline{G} \subseteq \underline{H} \,.\!\supset\!.\, \underline{a} \in \underline{H}$

as sole axiom schema for singular predication. Finally a formula of this logic is valid if and only if it is truth-functionally implied by these three axiom schemata or their alphabetic variants. Since no letters are relevant except those in the implicandum, a truth table affords a decision procedure.

This meager logic can be speciously enriched by a contextual definition of the relative clause. First let me say something about relative clauses. If a name of an object $\underline{a}$ occurs in an atomic sentence, '$\underline{a} \in \underline{F}$', what the sentence says about $\underline{a}$ is already nicely isolated in the predicable '$\underline{F}$', which is the predicate of the sentence. But sometimes we want also to isolate what a compound sentence says about $\underline{a}$; and such is the utility of the

relative clause. The name '$\underline{a}$' occurs buried in the sentence, perhaps in several places. The relative clause in English is formed from the sentence by putting 'which' for one of the occurrences of '$\underline{a}$', and 'it' for any subsequent occurrences, and inverting so as to bring 'which' forward. Thus from the sentence:

(4) White built Wit's End and Woollcott bought it (Wit's End) from him

we abstract the relative clause:

White built which and Woollcott bought it from him

or, rectifying the word order:

which White built and Woollcott bought from him.

This is a complex predicable, serving to segregate what (4) said about Wit's End. Predicating it of 'Wit's End', thus:

(5) Wit's End is (something) which White built and Woollcott bought from him

restores, in effect, the original sentence (4).

The inversion of word order, and in the above example the accompanying deletion of 'it', are idiomatic complications without logical value. A simpler alternative form for relative clauses is afforded by the 'such that' construction:

such that White built it and Woollcott bought it from him.

Relative clauses may occur nested, in the fashion:

> which Woollcott bought from a man who built it.

Regimented in the 'such that' idiom, this becomes:

> such that Woollcott bought it from (something) such that it is a man and it built it.

There ensues an ambiguity of cross-reference: 'it built it'. Often in ordinary language such ambiguities are obviated by differences in gender: 'he', 'she', 'it'. Sometimes not. The mathematician settles matters by invoking the mathematical counterpart of the pronoun, namely the bound variable:

> $\underline{x}$ such that Woollcott bought $\underline{x}$ from (something) $\underline{y}$ such that $\underline{y}$ is a man and $\underline{y}$ built $\underline{x}$.

This use in the role of relative pronoun is surely the basic use of the bound variable. One thinks first of quantification, but it is a mistake to do so. 'All $\underline{F}$ are $\underline{G}$' is well expressed by '$\underline{F} \subseteq \underline{G}$', and has been belabored since antiquity with no thought of bound variables. We can quantify simply by saying that everything is an $\underline{F}$, or that there are $\underline{F}$. The bound variable enters quantification only in order to construct the desired predicable '$\underline{F}$', when, as is usual, a complex predicable is wanted. The quantifications '$(\forall \underline{x})\phi\underline{x}$' and '$(\exists \underline{x})\phi\underline{x}$' are best seen as cases of '$\forall\underline{F}$' and '$\exists\underline{F}$', namely:

$$\forall(\underline{x} \text{ such that } \phi\underline{x}), \qquad \exists(\underline{x} \text{ such that } \phi\underline{x}),$$

in which the bound variable is just the relative pronoun, the

variable of the 'such that' construction. The bound variable does its distinctive work in segregating what a context says about something from what it says it about. We may want to segregate this material for any of various purposes: preparatory to saying that it holds of everything, or of something, thus quantification, or perhaps preparatory to framing a singular description or a class name. What is thus segregated, for whatever purpose, is the relative clause, and at this point the work of the bound variable is done.

The meaning of the relative clause is summed up in what I call the principle of <u>concretion</u>; namely, if we derive a relative clause from a sentence by abstracting from some name therein, and then we adjoin the resulting clause again to that name by predication, we are carried back in effect to the original sentence. The point was illustrated in the examples (4) and (5). This principle can be used to provide a contextual definition of the relative clause, within the special context of predication, thus:

$$\text{'}\underline{a} \in (\underline{x} \text{ such that } \phi\underline{x})\text{' for '}\phi\underline{a}\text{'}.$$

In particular this definition can be adopted in our meager logic of copulas and truth functions, to get back at last to that. However, just as in that logic I rendered the copulas as '$\in$' and '$\subseteq$', thus mimicking and anticipating set theory, so I shall render '$\underline{x}$ such that $\phi\underline{x}$' as '$\{\underline{x}: \phi\underline{x}\}$'; but we must keep in mind for a while that this expression is meant only as an innocent complex predicable, the relative clause 'which ϕ', 'such that it ϕ', '$\underline{x}$ such that $\phi\underline{x}$', and not as a class name.

Here, then, is our contextual definition by concretion

(6) $\quad$ '$a \in \{x: \phi x\}$' for 'ϕa'.

Of course 'ϕa' stands for any formula containing 'a', and 'ϕx' stands correspondingly. Usually in the predicate calculus the notations 'Fa' and 'Fx' serve this purpose, but I have reserved 'F', 'G', etc. to stand rather for free-standing predicables.

On the strength of the above definition, however, the letters 'F', 'G', etc. may hereafter stand for the new predicables '$\{x: \phi x\}$' as well as for primitive predicables. Those schematic letters thus regain, after all, the same versatility that they enjoyed in the usual predicate calculus. Any formula represented as 'ϕa' or 'ϕx' can just as well be represented hereafter as '$a \in F$' or '$x \in F$; for we can take 'F' to stand for '$\{x: \phi x\}$' whenever it occurs after '$\in$'.

The Boolean functions of predicables now become definable in obvious fashion.

$$\text{'}-F\text{' for '}\{x: \sim(x \in F)\}\text{'},$$
$$\text{'}F \cap G\text{' for '}\{x: x \in F \,.\, x \in G\}\text{'},$$
$$\text{'}F \cup G\text{' for '}\{x: x \in F \,.\!\vee\!.\, x \in G\}\text{'}.$$

For a half century and more it has been usual in the first-order predicate calculus to deprive oneself of these convenient notations in order not to presuppose an ontology of classes. This, we see, is a mistake; there is nothing afoot but the relative clause, a contextually definable predicable. And it is just these innocent Boolean functions, ironically, that go by the name of set theory in the "new math" of elementary schools.

Even in genuine set theory, with its explicit assumption of sets and perhaps ultimate classes as well, it is useful still to retain the non-committal relative clause and to represent it as here with a schematic 'F', 'G', etc. A conspicuous case of its utility is in formulating the axiom schema of replacement.[2] These are the places where Gödel appealed evasively to "notions"[3] and I, in a needless metaphor, to "virtual classes."[4] There need be no question of fictions, manners of speaking, somehow beyond sets and ultimate classes. It is a simple matter of schematic letters for complex predicables. I shall say more of this at the end.

Back now to our meager logic of copulas and truth functions. It is still only that, for all its specious new riches. The bound variable has emerged in it by contextual definition, but still no quantification. We could define '$(\forall x)(x \in F)$' as '$-F \subseteq F$' if this latter were defined, but it is not. '$-F$' is a relative clause '$\{x: \sim(x \in F)\}$', and relative clauses were defined in (6) only in predication: only after the singular copula '$\in$'.

The time is ripe for our first step of genuine augmentation. It consists simply in admitting relative clauses into categorical contexts; hence in allowing 'F' and 'G' henceforth to stand for relative clauses '$\{x: \phi x\}$' not only after '$\in$' but also in '$F \subseteq G$'. This is a genuine extension of our logic, since relative clauses alongside the plural copula '$\subseteq$' must be recognized as primitive notation; they are not eliminable by contextual definition. Quantification is now indeed definable:

$$\text{'}(\forall x)(x \in F)\text{' for '}-F \subseteq F\text{'}, \qquad \text{'}(\exists x)(x \in F)\text{' for '}\sim(F \subseteq -F$$

Our augmented logic amounts precisely to the lower monadic predicate calculus; for not only do we now have quantification, but also, conversely, '$F \subseteq G$' is definable in the predicate calculus, viz. as '$(\forall x)(Fx \supset Gx)$'.

I have long held that the bound variable — hence the relative pronoun — is the key to objective reference. The objects of a theory are the values of its bound variables. The subject-predicate distinction is derivative: the singular subject of a sentence is the part that stands where a bound variable could stand, and the predicate is the rest. Thus it seems fair to say that our initial logic of copulas and truth functions was pre-referential. When I spoke of 'a' and 'F' as standing for a name and a predicable, and as subject and predicate in '$a \in F$', I was anticipating their status in an augmented logic. Of themselves all that can properly be said is that they combine to form a sentence, true or false; they are no more subject and predicate than predicate and subject.

The anticipation became more pointed when I introduced the bound variable by contextual definition. On the face of it the bound variable purports to take objects as values, and on the fact of it 'a' and 'F' thus come into their own as subject and predicate, name and predicable. But I say "on the face of it" and "purports" because the bound variable entered here only by a gratuitous and eliminable notational trick; it is not in the primitive apparatus.

The move to the augmented logic changes this. Now that we are admitting relative clauses alongside the plural copula, the

bound variable must be taken seriously; it is primitive, in those contexts, and not eliminable. We move from a logic of simulated reference to a logic genuinely of reference — indeed, as noted, the lower monadic predicate calculus. The letters 'a' and 'F' stand for subject and predicate, name and predicable, by mere anticipation or simulation no longer; the anticipated time is come and their status is clinched.

At the same time, ironically enough, the schematic name letters 'a', 'b', etc. become superfluous. We can gain their generality explicitly through universal quantification, now that it is defined. E.g. the axiom schema (3) can now give way to this:

$$(\forall x)(x \in G \;.\; G \subseteq H \;.\supset.\; x \in H).$$

We can leave the initial quantifier tacit or, better, we can strengthen the schema to read:

$$G \subseteq H \;.\equiv\; (\forall x)(x \in G \;.\supset.\; x \in H).$$

This is not a definition, since we used '$\subseteq$' in defining '$(\forall x)$'.

In our augmented logic we can now indulge ourselves in a second-level simulation, introducing bound variables for classes by contextual definition:

(7) $$\text{'}F \in \{x^1 : \phi x^1\}\text{' for '}\phi F\text{'}.$$

The bound variable 'x^1' purports to take classes as values, and on the face of it the old predicables become names of classes. But again it is a manner of speaking, an eliminable notational trick.

So we now have second-level relative clauses, '$\{\underline{x}^1: \phi\underline{x}^1\}$', whose relative pronouns purport to take classes as values. We proceed in turn to their Boolean functions:

$$\text{'}-\{\underline{x}^1: \phi\underline{x}^1\}\text{'} \quad \text{for} \quad \text{'}\{\underline{x}^1: \sim\phi\underline{x}^1\}\text{'},$$

$$\text{'}\{\underline{x}^1: \phi\underline{x}^1\} \cap \{\underline{y}^1: \psi\underline{y}^1\}\text{'} \quad \text{for} \quad \text{'}\{\underline{x}^1: \phi\underline{x}^1 \,.\, \psi\underline{x}^1\}\text{'},$$

and similarly for the union. Our simulated class variables can occur wherever the old predicables can – hence after 'ϵ' and before and after '$\subseteq$'. But not yet in quantifiers. We could define '$(\forall\underline{x}^1)\phi\underline{x}^1$' as '$-\{\underline{x}^1: \phi\underline{x}^1\} \subseteq \{\underline{x}^1: \phi\underline{x}^1\}$' if this latter were defined, but it is not; second-level relative clauses were defined in (7) only in the position after 'ϵ'.

Accordingly the time is ripe for our second step of genuine augmentation: admittance of second-level relative clauses alongside '$\subseteq$'. Quantification of class variables becomes definable: universal as above and existential correspondingly. We have risen to the second-order monadic predicate calculus, which is to say the lowest level of set theory: the theory of classes of individuals. The bound class variable must be taken seriously now; it is primitive, in the categorical contexts, and not eliminable. Classes have unequivocally been hypostatized, and predicables have come unequivocally to name them. It is just here that the subtle transition from logic to set theory has taken place. It comes not when we contextually define the second-level relative clause with its ostensible class variable, but only when we allow such clauses to stand irreducibly alongside '$\in$'. It is only then that '$\underline{F}$', '$\underline{G}$', and '$\{\underline{x}: \phi\underline{x}\}$' cease to stand for predicables and come to stand genuinely for class names; and it is only then that '$\underline{x} \in \underline{F}$' and '$\underline{F} \subseteq \underline{G}$' come to express class membership and inclusion.

At the same time the schematic letters '$\underline{F}$', '$\underline{G}$', etc. become superfluous. We can gain their generality explicitly through universal quantification over classes, now that such quantification has been defined. E.g., the axiom schema (3) can now give way to this:

$$(\forall\underline{z}^1)(\forall\underline{y}^1)(\forall\underline{x})(\underline{x} \in \underline{y}^1 \,.\, \underline{y}^1 \subseteq \underline{z}^1 \;.\supset.\; \underline{x} \in \underline{z}^1).$$

We can leave the three quantifiers tacit if we like or, better, we can strengthen the formula to read:

$$\underline{y}^1 \subseteq \underline{z}^1 \;.\equiv\; (\forall\underline{x})(\underline{x} \in \underline{y}^1 \;.\supset.\; \underline{x} \in \underline{z}^1).$$

The schematic letters '$\underline{a}$', '$\underline{b}$', etc. and '$\underline{F}$', '$\underline{G}$', etc. that were needed in expounding the initial logic of copulas and truth functions are now retired; '$\underline{a}$', '$\underline{b}$', etc. ceased to be needed after the first augmentation of logic, and '$\underline{F}$', '$\underline{G}$', etc. after the second. We still need 'ϕ', 'ψ', etc. to schematize higher-order contexts; we needed them in communicating the last three definitions.

The cycle that we have now observed twice can be continued. We are now in the third system of our hierarchy. Continuing the pattern, we next enrich this system speciously by a contextual definition of third-level relative clauses, whose relative pronouns simulate class variables of second type.

(8) $\quad \text{'}\{\underline{x}^1: \phi\underline{x}^1\} \in \{\underline{y}^2: \psi\underline{y}^2\}\text{'} \quad \text{for} \quad \text{'}\psi\{\underline{x}^1: \phi\underline{x}^1\}\text{'}.$

Again we get Boolean functions:

$$`-\{\underline{x}^2: \phi \underline{x}^2\}' \text{ for } `\{\underline{x}^2: \sim\phi \underline{x}^2\}'$$

and so forth, but not yet '$(\forall \underline{x}^2)$'. A third step of genuine augmentation is thus invited, allowing third-level relative clauses to occur irreducibly alongside '$\subseteq$'. Thereupon we can define:

$$`(\forall \underline{x}^2)\phi \underline{x}^2' \quad \text{for} \quad `-\{\underline{x}^2: \phi \underline{x}^2\} \subseteq \{\underline{x}^2: \phi \underline{x}^2\}'$$

and correspondingly for '$(\exists \underline{x}^2)$'. Classes of second type are now genuinely hypostatized; we have risen to third-order monadic predicate calculus, or the theory of classes of sets of individuals. And so we may continue up the hierarchy of types. Each step upward is achieved not by the contextual definition of the next type of relative clause, but by admitting those clauses alongside '$\subseteq$'.

What is thus generated is the simple classical theory of types. It does not exclude classes that have to be specified <u>impredicatively</u>. An expression '$\{\underline{x}^n: \phi \underline{x}^n\}$' is impredicative if the membership condition represented as '$\phi \underline{x}^n$' contains bound variables of type higher than $\underline{n}$ or free ones of type higher than $\underline{n} + 1$. Example:

$$(9) \qquad \{\underline{x}: (\exists \underline{y}^1)(\underline{x} \in \underline{y}^1 \,.\, \underline{y}^1 \in \underline{z}^2)\}.$$

The reason such impredicative membership conditions are accommodated is that each of the contextual definitions (6), (7), (8), and their suite is retained for further use in ensuing systems. Thus in our initial logic of copulas and truth functions, where the

form '$\{\underline{x}: \phi \underline{x}\}$' was defined contextually by (6), the clause (9) makes no sense; the variables '$\underline{y}^1$' and '$\underline{z}^2$' are foreign notation, and so is existential quantification. In the third system, however, those notations are all at hand, and the old definition (6) is still standing by to be applied again, yielding (9).

It is important that impredicative membership conditions become available, since some of the classes required for the foundations of classical analysis are not otherwise specifiable. A familiar place where this need arises is in proving that every bounded set of real numbers has a least bound; and indeed (9) itself is the construction that is called for at that point.

Russell attributed his paradox and other antinomies to violations of what he called the vicious-circle principle. It says roughly that the definition of a class must not refer to any range of objects which already includes that class or other classes whose definitions depend on that class. The theory of types was his way of implementing this principle. But he worried about impredicative membership conditions, which, though needed for classical analysis, seem to violate the principle. Hence his resort to his untidily ramified theory of types and his <u>ad hoc</u> axiom of reducibility. Better to discard the vicious-circle principle itself in favor of some more liberal intuition, such as might lend intuitive support outright to the simple theory of types with all its tolerance of impredicative membership conditions.

Perhaps the series of augmentations that we have now been studying has some merit in that regard. One merit lies in the naturalness and seeming slightness of the successive steps of augmentation; each consists merely in allowing clauses already admissible after 'ϵ' to occur also beside '$\subseteq$'. It is a continuing series of steps the first of which reifies not classes but the individuals themselves. Another merit is that the meaninglessness of formulas that mix types is not just arbitrarily decreed, but becomes more a matter of absence of definition.

I have been adhering to the monadic. Once we have risen to classes $\underline{x}^2$ of second type, there are familiar ways of defining ordered pairs of individuals and thus transcending the monadic. Still, if our hierarchy of systems is to be viewed somewhat in the spirit of a fictional essay in the psychogenesis of logic and set theory, it would be more realistic to admit polyadic logic at the beginning, thus assuming polyadic predicables and the corresponding copulas. Then, once we had ascended to classes $\underline{x}^2$ of second type, we would economize retroactively by dropping the primitive polyadic appurtenances and defining ordered pairs to take up the burden. The theory of types is incomparably simpler when so based than when rested on irreducibly polyadic beginnings.

The constructions that we have been contemplating provide for the existence of all classes for which we can formulate membership conditions conforming to the theory of types. But there are also non-constructive axioms to consider, ones that postulate classes for which in general no full membership conditions are available. One is the axiom of infinity, which postulates an infinite class; another is the axiom of choice, which postulates selection classes; another is the continuum hypothesis, which postulates certain mappings. We know from Gödel's theorem that

there is no end of room for supplementation. As usual, such axioms would be added as desired, but presumably the axiom of infinity is the only addition needed for applied mathematics.

Our hierarchy of systems develops the simple theory of types in its classical form, with mutually exclusive types. Cumulative types are less confining. Our construction could be revised at each stage, at no great cost, so as to issue rather in cumulative types. To the definition (7) of '$\{\underline{x}^1 : \phi\underline{x}^1\}$' we would add an echo of (6), thus:

'$\underline{F} \in \{\underline{x}^1 : \phi\underline{x}^1\}$' for '$\phi\underline{F}$', '$\underline{y} \in \{\underline{x}^1 : \phi\underline{x}^1\}$' for '$\phi\underline{y}$',

appreciating that for any particular formula in the role of '$\phi\underline{x}^1$' only one of the two definientia would be grammatically coherent. To the definition (8) of '$\{\underline{x}^2 : \phi\underline{x}^2\}$', similarly, we would add echoes of (7) and (6); and so on up.

An attractive departure from the cumulative theory of types, in turn, is its transcription into a notation of unrestricted variables, dispensing with type indices. Ranging over all types as they do, the new variables render previously meaningless formulas meaningful. Additional axioms are thus invited, to deny what could not be denied before. We can no longer say, on pain of the antinomies, that all classes exist for which we can formulate existence conditions; rather we must marshal explicit constraints. Natural adjustments in this spirit carry us in turn from the cumulative theory of types, with unrestricted variables, to Zermelo's set theory. After all this, a further departure that brings rewards is the hypostasis of ultimate classes:

classes that are members of none. Outside this sequence of departures also there are other systems of set theory, more radical alternatives, that bear some interest.[5] But amid all this variety the simple theory of types may perhaps be looked upon as the nuclear set theory, the nearest to nature. The rest are departures deliberately fashioned with an eye to elegance, strength, or convenience.

While we generated the successive types, the relative clauses of successive levels became names of classes of successive types. The singular and plural copulas came thereby to be predicables in their own right, the two-place predicables of membership and inclusion. There ceased to be relative clauses as distinct from class names. When we turn from the theory of types to any of the systems of set theory with unrestricted variables, on the other hand, the situation is different, since membership conditions there become formulable for which there are no classes. For such systems the relative clause retains relevance; it may or may not prove to name a class, and it can be useful still when it does not. As I mentioned earlier, it does its simple old work where Gödel once talked of notions and I of virtual classes.

We can incorporate relative clauses very smoothly into the formalism of such systems, retaining the same notation '$\{\underline{x}: \phi \underline{x}\}$' for them whether or not there is a corresponding class. When there is, the relative clause counts as a name of it; but it remains a relative clause in either event. We might view matters as follows: relative clauses were with us as predicables from antiquity, and only in our later sophistication we posited abstract objects, classes, for some of them to do double duty as names of. Regrettably we could not posit classes for all of them, on pain of contradiction. When the clause does name a class, however, we know how to say so:

$$(\exists \underline{y})(\underline{y} = \{\underline{x}: \phi \underline{x}\}).$$

It is only on such an existence premiss that we are entitled to substitute the clause for a quantified variable or vice versa in logical inference by universal instantiation or existential generalization. We have a case here, as Dana Scott remarked, of free logic. He was referring to my combined use of virtual and real classes in Set Theory and Its Logic, and that is indeed just what I have been describing in very different terms in the present paragraph.

July 21, 1976.

FOOTNOTES

[1]I intend nothing by predicables other than what we in logic persist in calling predicates, but I feel moved to support Geach in his restitution of the useful traditional distinction between linguistic form and grammatical function. The predicable is the expression itself, and this expression is the predicate of, or in, any sentences in which it is predicated.

[2]See Quine, The Ways of Paradox and Other Essays, enlarged edition (Cambridge: Harvard, 1976), p. 279.

[3]Gödel, The Consistency of the Continuum Hypothesis, Princeton, 1940.

[4]Quine, Set Theory and Its Logic, Cambridge: Harvard, 1963, 1969.

[5]See Set Theory and Its Logic, Chapters XII-XIV, on all these points.

[6]Dana Scott, "Existence and description in formal logic," in R. Schoenman, ed., Bertrand Russell, Philosopher of the Century (London: Allen and Unwin; Boston: Little, Brown; 1967), pp. 181-200.